Imaging fast moving objects with application to satellite imaging

George Papanicolaou
Stanford University

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With: J. Fournier, J. Garnier, C. Tsogka and L. Borcea, K. Solna,

How did this research project start

Use passive sensor arrays as a way to image (a) through strongly inhomogeneous media, and (b) with independent, asynchronous, and unknown (often opportunistic) illumination.

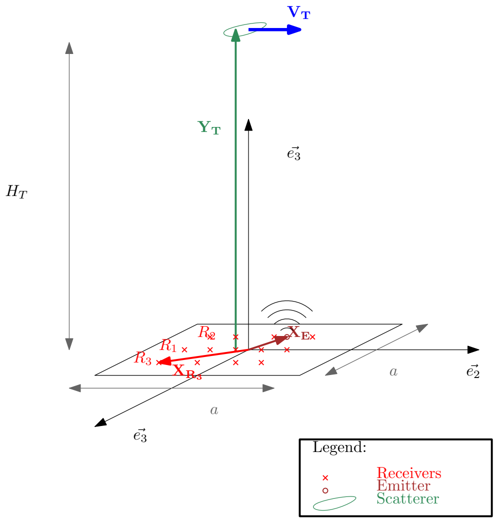
1. Started in seismic imaging for hydrocarbons around 2005 but has a longer history. Around 2010 passive synthetic aperture radar (SAR) begun to be used for imaging ground reflectivities using opportunistic illumination, either ground based on from satellites[1].
2. We decided to consider passive SAR to image satellites. The passive recording platform(s) is (are) to fly above the atmosphere (at about 20 km or more), the illumination coming from the ground. The satellite is low flying (at about 300-1000 km), and is rapidly moving (to remain in orbit).

[1] Passive Imaging with Ambient Noise, Garnier and Papanicolaou, Cambridge University Press 2016

What can theory and numerical simulations do

1. The effect of atmospheric inhomogeneities is minimized with high-flying, passive receivers (Garnier+P, SIIMS 2014 and 2015).
2. Focus the theory on resolution analysis. Analytically challenging but can be done from first principles.
3. Compare ground-based matched-filter imaging (currently in use) and (the proposed) passive receiver, correlation-based imaging.
4. Main result: In the X-Band (10 GHz) regime, and with six to nine recording platforms (ground-based or drones) over a 200×200 kilometer region the satellite position and velocity image resolutions are comparable for the two modalities, can be quantified very well, and are close to optimal, down to centimeter level (with a wavelength of 3cm).

Imaging with passive auxiliary arrays schematic



Configuration with receivers above ground

Scaterring by a moving object

A (point) transmitter at $\boldsymbol{X}_{\mathsf{E}}$ emits a short pulse $f(t)$. The total field $u(t, \boldsymbol{x})$ solves

$$\frac{1}{c^2(t,\boldsymbol{x})}\frac{\partial^2 u}{\partial t^2} - \Delta u = f(t)\delta(\boldsymbol{x} - \boldsymbol{X}_{\mathsf{E}}), \tag{1}$$

with a localized perturbation ρ_{T} centered at the moving target $\boldsymbol{X}_{\mathsf{T}}(t)$,

$$\frac{1}{c^2(t,\boldsymbol{x})} = \frac{1}{c_o^2}\Big(1 + \rho_{\mathsf{T}}\big(\boldsymbol{x} - \boldsymbol{X}_{\mathsf{T}}(t)\big)\Big).$$

The incident field $u^{(0)}(t, \boldsymbol{x})$ is

$$u^{(0)}(t,\boldsymbol{x}) = \frac{1}{4\pi|\boldsymbol{x} - \boldsymbol{X}_{\mathsf{E}}|} f\Big(t - \frac{|\boldsymbol{x} - \boldsymbol{X}_{\mathsf{E}}|}{c_o}\Big). \tag{2}$$

The scattered field

In the Born approximation the scattered field is given by

$$u^{(1)}(t,\boldsymbol{x}) = -\frac{1}{c_o^2}\int_0^t d\tau \int d\boldsymbol{y} G(t-\tau,\boldsymbol{x},\boldsymbol{y})\rho_T(\boldsymbol{y}-\boldsymbol{X}_T(\tau))\frac{\partial^2}{\partial\tau^2}u^{(0)}(\tau,\boldsymbol{y}).$$

For a point-like scatterer,

$$u^{(1)}(t,\boldsymbol{x}) = -\frac{\rho}{c_o^2}\int_0^t d\tau G(t-\tau,\boldsymbol{x},\boldsymbol{X}_T(\tau))\frac{\partial^2}{\partial\tau^2}u^{(0)}(\tau,\boldsymbol{y})\mid_{\boldsymbol{y}=\boldsymbol{X}_T(\tau)},$$

where $\rho = \int \rho_T(\boldsymbol{x})d\boldsymbol{x}$ is the reflectivity of the target. Using $u^{(0)}$ and integrating by parts twice:

$$u^{(1)}(t,\boldsymbol{x}) = -\frac{\rho}{c_o^2}\int_0^t d\tau \int_0^\tau d\tau' f''(\tau')G(\tau-\tau',\boldsymbol{X}_T(\tau),\boldsymbol{X}_E)G(t-\tau,\boldsymbol{x},\boldsymbol{X}_T(\tau)).$$

Therefore the scattered field at the receiver at $\boldsymbol{x}=\boldsymbol{X}_R$ is

$$\begin{aligned} u_{s,R}(t) &= -\frac{\rho}{c_o^2}\int_0^t d\tau \frac{1}{4\pi|\boldsymbol{X}_T(\tau)-\boldsymbol{X}_E|} f''\Big(\tau - \frac{|\boldsymbol{X}_T(\tau)-\boldsymbol{X}_E|}{c_o}\Big) \\ &\quad \times \frac{1}{4\pi|\boldsymbol{X}_R-\boldsymbol{X}_T(\tau)|}\delta\Big(t-\tau-\frac{|\boldsymbol{X}_R-\boldsymbol{X}_T(\tau)|}{c_o}\Big). \end{aligned}$$

The scattered field, continued

If we introduce

$$\Phi(\tau;t) = t - \tau - \frac{|\boldsymbol{Y}_\mathsf{T} - \boldsymbol{X}_\mathsf{R} + \tau \boldsymbol{V}_\mathsf{T}|}{c_o},$$

then we have

$$\delta\left[\Phi(\tau;t)\right] = \frac{\delta[\tau - \tau(t)]}{|\partial_\tau \Phi(\tau(t);t)|},$$

with $\tau(t)$ the unique zero of $\tau \to \Phi(\tau;t)$ in $(0,t)$. Denoting $\boldsymbol{D}(t) = \boldsymbol{Y}_\mathsf{T} - \boldsymbol{X}_\mathsf{R} + t\boldsymbol{V}_\mathsf{T}$, We find that $\tau(t)$ is given by

$$\tau(t) = t - \frac{|\boldsymbol{D}(t)|}{c_o\left(1 - \left|\frac{\boldsymbol{V}_\mathsf{T}}{c_o}\right|^2\right)} \left[\sqrt{1 - \left|\frac{\boldsymbol{V}_\mathsf{T}}{c_o}\right|^2 + \left(\frac{\boldsymbol{V}_\mathsf{T}}{c_o} \cdot \frac{\boldsymbol{D}(t)}{|\boldsymbol{D}(t)|}\right)^2} - \frac{\boldsymbol{V}_\mathsf{T}}{c_o} \cdot \frac{\boldsymbol{D}(t)}{|\boldsymbol{D}(t)|}\right]. \tag{3}$$

Using this in $u_{\mathsf{s},\mathsf{R}}(t)$ we get the (model) signal recorded at the receiver

$$u_{\mathsf{s},\mathsf{R}}(t) = -\frac{\rho f''\left(\tau(t) - \frac{|\boldsymbol{X}_\mathsf{T}(\tau(t)) - \boldsymbol{X}_\mathsf{E}|}{c_o}\right)}{(4\pi)^2 c_o^2 |\boldsymbol{X}_\mathsf{T}(\tau(t)) - \boldsymbol{X}_\mathsf{E}||\boldsymbol{X}_\mathsf{R} - \boldsymbol{X}_\mathsf{T}(\tau(t))| \left|1 + \frac{\boldsymbol{V}_\mathsf{T}}{c_o} \cdot \frac{\boldsymbol{D}(\tau(t))}{|\boldsymbol{D}(\tau(t))|}\right|}. \tag{4}$$

What is the imaging problem

- We record $u_{s,R}(t)$ at various receiver locations $\boldsymbol{X}_R$. These locations (not moving for simplicity here) are assumed known.
- The source location must be know in matched field imaging. It need not be known for correlation based imaging.
- We want to find (estimate) the target location $\boldsymbol{Y}_T$ and velocity $\boldsymbol{V}_T$. This is a point in six dimensions in general. For satellites it can be reduced to five with a "tangential" $\boldsymbol{V}_T$.

How are we to do this? We construct Imaging functions.

Imaging functions: Matched field

The idea behind the matched-filter imaging function is that we want to match the received signal with the emitted pulse. The matching process involves the assumed initial position and speed of the object $(\boldsymbol{Y}, \boldsymbol{V})$, and this matching can be shown to be maximal at the true position $(\boldsymbol{Y}_\mathsf{T}, \boldsymbol{V}_\mathsf{T})$. The matching process takes into account a (derived) Doppler compensation factor $\gamma_\mathsf{s}(\boldsymbol{X}, \boldsymbol{V}, \boldsymbol{X}_\mathrm{R})$,

$$\mathcal{I}^{\mathsf{MF}}(\boldsymbol{Y}, \boldsymbol{V}) = \frac{1}{N_E} \sum_{j=1}^{N_E} \mathcal{I}_j^{\mathsf{MF}}(\boldsymbol{Y} + \boldsymbol{V} S_j, \boldsymbol{V}),$$

$$\mathcal{I}_j^{\mathsf{MF}}(\boldsymbol{X}, \boldsymbol{V}) = \frac{1}{N} \sum_{R=1}^{N} \int f\Big(\gamma_\mathsf{s}(\boldsymbol{X}, \boldsymbol{V}, \boldsymbol{X}_\mathrm{R})\big(t - \frac{|\boldsymbol{X} - \boldsymbol{X}_\mathrm{R}|}{c_o}\big) - \frac{|\boldsymbol{X} - \boldsymbol{X}_\mathrm{E}|}{c_o}\Big)$$
$$u_{\mathsf{s},R}(S_j + t)dt$$

This imaging function requires knowledge of the transmitter and receiver positions $\boldsymbol{X}_\mathrm{E}$ and $\boldsymbol{X}_\mathrm{R}$. We also need to know the pulse profile f. One wants to image a region around some point $\boldsymbol{Y}_\mathsf{T}$, so the j-th scattered signal needs only to be recorded for a short time around $2|\boldsymbol{Y}_\mathsf{T} - \boldsymbol{X}_\mathrm{E}|/c_o$.

Imaging functions: Cross correlations

We cross correlate the scattered signals recorded by pairs of receivers and migrate them with the appropriate Doppler compensation factors,

$$\mathcal{I}^{\mathrm{CC}}(\boldsymbol{Y},\boldsymbol{V}) = \frac{1}{N_E}\sum_{j=1}^{N_E}\mathcal{I}_j^{\mathrm{CC}}(\boldsymbol{Y}+\boldsymbol{V}S_j,\boldsymbol{V}), \tag{5}$$

$$\begin{aligned}\mathcal{I}_j^{\mathrm{CC}}(\boldsymbol{X},\boldsymbol{V}) = &\frac{1}{N^2}\sum_{R,R'=1}^{N}\int u_{s,R}\Big(S_j+\frac{|\boldsymbol{X}-\boldsymbol{X}_R|}{c_o}+\frac{t+\frac{|\boldsymbol{X}-\boldsymbol{X}_E|}{c_o}}{\gamma_s(\boldsymbol{X},\boldsymbol{V},\boldsymbol{X}_R)}\Big)\\ &\times u_{s,R'}\Big(S_j+\frac{|\boldsymbol{X}-\boldsymbol{X}_{R'}|}{c_o}+\frac{t+\frac{|\boldsymbol{X}-\boldsymbol{X}_E|}{c_o}}{\gamma_s(\boldsymbol{X},\boldsymbol{V},\boldsymbol{X}_{R'})}\Big)dt.\end{aligned} \tag{6}$$

Bow it is not necessary to know the pulse profile f which could be different from one emission to another one. It is not necessary either to know the emission times with accuracy. But we need to record the whole train of scattered signals. Moreover correlation-based imaging has been shown to be robust to medium fluctuations when in a suitable imaging configuration[2].

[2] Garnier+P, CUP, 2016

Simplified setup for the simulations

- We assume that there is a single illuminating source on the ground, whose location need not be known for CC imaging. The emitted signals (synchronization, pulse form) are also not known. They are, however assumed known for MF imaging.
- The 6-9 recording platforms are stationary (as their motion makes little difference in resolution if assumed known) and randomly placed in a 200×200 kilometer square at a fixed altitude. The satellite flies in the Y_2 direction (into the screen) at constant speed starting right above the source on the ground.
- With only about 6-9 recording platforms we get as good a resolution as if we had a full 200×200 kilometer aperture. Both with CC and MF imaging.

Satellite imaging with (passive) X-band SAR

System Parameters		
Central Frequency	f_0	9.6 GHz
Bandwidth	B	622 MHz
Number of Frequencies in Bandwidth	N_f	515
Slow-time Sampling	Δs	0.015 s
Wave Speed	c_o	3×10^8 m/s
Central Wavelength	λ_o	3.12 cm
Altitude of Satellite	H	500 km
Speed of Satellite	V_T	7,610.6 m/s
Altitude of Drone	h	20 km
Velocity of Drone	V_R	222.2 m/s (800 km/hr)

Parameters for modeling SAR imagining of a satellite with passive SAR on a platform above the atmosphere and microwave sources on the ground.

Resolution results from the simulation

- There are five "parameters" to be imaged: The three components of the satellite (say its initial) location and the (assumed) two components of its speed. We actually include vertical speed as well since it is needed when dealing larger size space objects.
- The passive SAR platform covers a distance of 5 km, in 22.5 secs. During this time the satellite covers a distance of 171 km. These are the recording windows used.

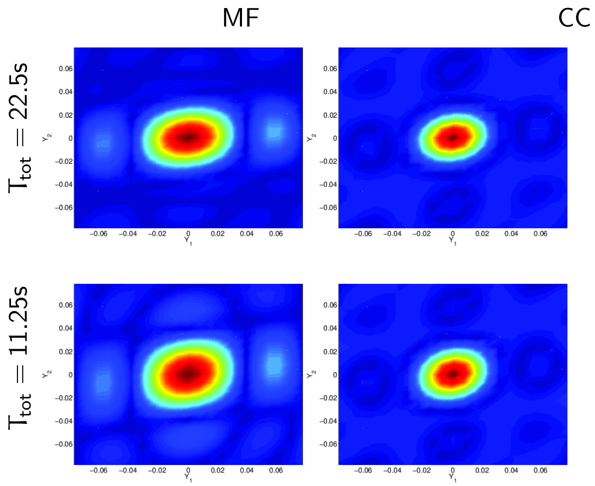
MF theoretical imaging resolution formulas obtained

Y_1	$\frac{\lambda_o H_T}{a}$	9 cm
Y_2	$\lambda_o \left(\frac{H_T}{a} \wedge \frac{H_T}{2\lvert V_T\rvert_\perp T_{tot}}\right)$	$9 \text{ cm} \wedge 9.7 \text{ cm} = 9 \text{ cm}$
Y_3	$\frac{c_o}{2B} \wedge \lambda_o \frac{H_T^2}{2\lvert V_T\rvert_\perp T_{tot} a}$	$25 \text{ cm} \wedge 29 \text{ cm} = 25 \text{ cm}$
V_1	$\frac{\lambda_o H_T}{a T_{tot}}$	$8.2\ 10^{-3}\ m.s^{-1}$
V_2	$\frac{\lambda_o}{T_{tot}} \left(\frac{H_T}{a} \wedge \frac{H_T}{2\lvert V_T\rvert_\perp T_{tot}}\right)$	$8.2\ 10^{-3}\ m.s^{-1} \wedge 8.9\ 10^{-3}\ m.s^{-1} = 8.2\ 10^{-3}$
V_3	$\frac{\lambda_o}{2T_{tot}}$	$1.4\ 10^{-3}\ m.s^{-1}$

CC theoretical imaging resolution formulas obtained

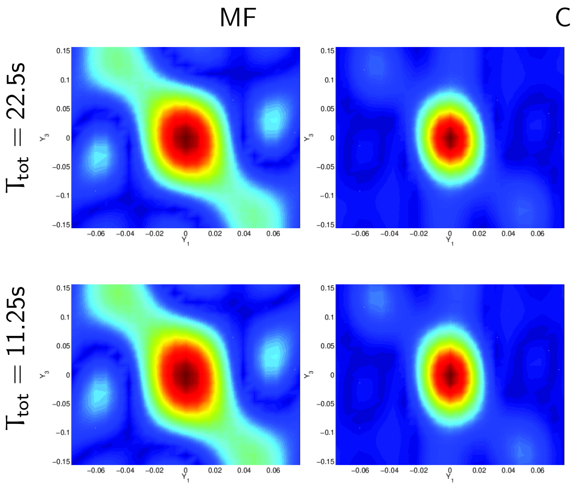
$\boldsymbol{Y}_\perp$	$\dfrac{\lambda_o H_T}{a}$	$1.4\ 10^{-3}m$
Y_3	$\lambda_o\left(\dfrac{H_T^2}{a^2} \wedge \dfrac{2H_T^2}{aV_T T_{tot}}\right)$	$9.0\ 10^{-2}m. \wedge 1.2m$
$\boldsymbol{V}_\perp$	$\dfrac{\lambda_o H_T}{aT_{tot}}$	$8.2\ 10^{-3}m.s^{-1}$
V_3	$\dfrac{\lambda_o}{T_{tot}}\left(\dfrac{H_T^2}{a^2} \wedge \dfrac{2H_T^2}{aV_T T_{tot}}\right)$	$2.5\ 10^{-2}m.s^{-1} \wedge 1.1\ 10^{-1}m.s^{-1}$

MF and CC horizontal-horizontal (Y_1, Y_2) resolutions



Images with MF and CC in the (Y_1, Y_2) plane. The units are in m. The satellite velocity is $V_T = 7610m/s$. The first row is for recording duration $T_{tot} = 11.25s$ and the second for $T_{tot} = 22.5s$.

MF and CC horizontal-vertical (Y_1, Y_3) resolutions



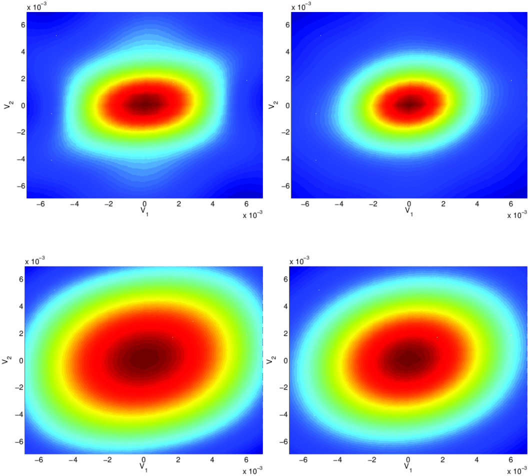
Images with MF and CC in the plane (Y_1, Y_3). The units are in m. The satellite velocity is $V_T = 7610$m/s. The first row is for recording duration $T_{tot} = 11.25$s and the second for $T_{tot} = 22.5$s.

MF and CC horizontal-horizontal (V_1, V_2) velocity resolutions

MF CC

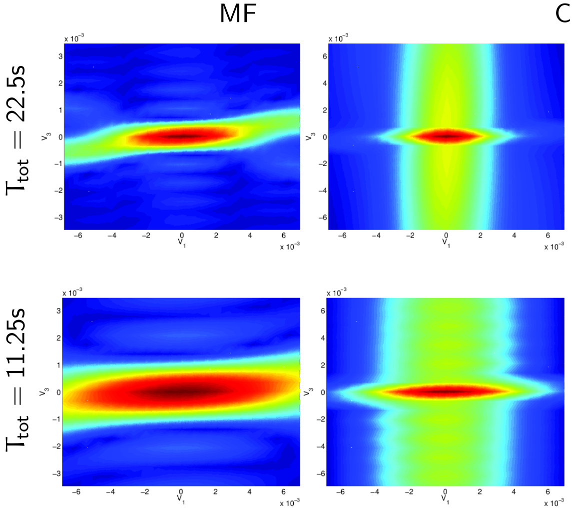
$T_{tot} = 22.5s$

$T_{tot} = 11.25s$



The abscissa is for V_1 and the ordinate for V_2. The units are in m/s. The

MF and CC horizontal-vertical (V_1, V_3) velocity resolution



The abscissa is for V_1 and the ordinate for V_3. The units are in m/s. The satellite velocity is $V_T = 7610$m/s. The first row is for recording duration $T_{tot} = 11.25$s and the second for $T_{tot} = 22.5$s.

Conclusions

- We have shown that passive SAR imaging of satellites can be done with a resolution that is essentially the optimal one, properly interpreted, when using a suitably adjusted imaging function to account for rapid target motion. The resolution theory is challenging but essentially complete now, both for CC and MF (currently used) imaging. CC and MF imaging resolutions are comparable for multiple receivers (continuum approximation) and "large" apertures[3].
- CC imaging is robust to atmospheric inhomogeneities when for example the satellite is low in the horizon and signal paths are long inside the atmosphere. Numerical simulations to explore this need to be done and are challenging.
- Need to address: Synchronization issues, SNR issues, finite size satellite effects, including rotation, swarms of debris, global small scale tracking .... Sparse Arrays ...

[3] Two papers in the SIAM J. on Imaging Science, 2017